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Abstract: The idea relying on attached culture for microalgae production has attracted many interest these past years due to their energy efficiency and low water usage. Microalgae can grow and attach to the surface of an appropriate material to form a biofilm. In this paper, a rotating algal biofilm (RAB) model is introduced. It is based on the Han model. How light affects the growth and productivity of microalgae and thus the formed biofilm will be discussed through model development, more importantly, it will be seen that taking into consideration light dilution factor can increase productivity. The benefit of the system is assessed when the conveyer velocity is fast enough. Simulation show an optimal folding of the conveyer. Actual productivities for moderate velocities are assessed and compared to these extreme cases.

Keywords: microalgae, rotating algal biofilm, biomass harvest

1. INTRODUCTION

Microalgae are considered for their potential use as a feedstock, not only for biofuel production (Hu et al. (2008), Moheimani et al. (2015)) but for a multitude of industries ranging from pharmaceutics to aquaculture (Benemann (1992), Borowitzka (1995)). They can also contribute to recycle nitrogen and phosphorus within a wastewater treatment process. A keen interest in attached microalgae culture has been shown these past years. Flemming and Wingender (2010) define a biofilm as microorganisms that live in a self-produced matrix of extracellular polymeric substances (EPS). The latter are mainly polysaccharides, proteins, nucleic acids and lipids, all constitute a three dimensional polymer network that interconnects biofilm cells. The main advantage on using rotating algal biofilm (RAB) reactors is the reduced cost for harvesting, and the high productivity due to light dilution in time. Indeed, cells are never too long in sunlight, so that photo inhibition (damages due to excess of light energy), is mitigated. The time microalgae are exposed to light affect their growth, that is, after a longer exposition to high light intensity, the cells become photo-saturated and often inhibited. Photoinihibition is characterized by the denaturation of some key proteins contributing to the photosynthetic activity. RAB offer the possibility to regulate light distribution through the biofilm by varying the rotational speed and therefore changing light exposure. The rotation also, allows the biofilm to be in permanent contact with the medium for the microalgae to survive and contribute to the biofilm formation. Christenson and Sims (2012) developed a RAB reactor for maximizing algae production in waste water treatment process, while reducing nitrogen and phosphorus concentration. Gross and Wen (2014) showed that RAB can provide, on average (over a year period), 302% increase in biomass productivity when compared to a standard raceway pond. Blanken et al. (2014) introduced a bioreactor they refer to as Algadisk. They showed that a productivity of $20 g.m^{-2}.d^{-1}$ can be obtained with *Chlorella Sorokiniana*. Schnurr et al. (2014) studied the effect of light direction on algal biofilm growth rate. It was concluded that light direction had no effect on long-term algal biofilm growth as opposed to its effect on suspended algal cells present in the growth medium. Wang et al. (2017) summarize the recently developed biofilm based attached cultivation technology. All these RAB photobioreactor benefit from the simple harvesting procedure consisting in scrapping the biomass out of the formed biofilm to avoid expensive sedimentation and centrifugation operations.

Few authors tackled the challenging issue of modeling such processes. In this work, we propose a model that would account for the effect of photoinhibition and respiration. We start from a model by Han (2002), that was firstly introduced by Eilers and Peeters (1988). After some approximation we augment it with the the biofilm thickness dynamics together with considering light attenuation through the layers of the formed bioflim.
The second part of the paper focuses on light distribution of the RAB, and how it affects the productivity. It is important to know that an exposure to high light intensity renders the microalgae inhibited, affecting the growth, and inherently the biofilm. A longer exposition to darkness however, decreases the carbon content of the cells by the effect of respiration. Therefore, a good compromise

⋆ Ademe PhytoRecolt project and IPL algae in silico.
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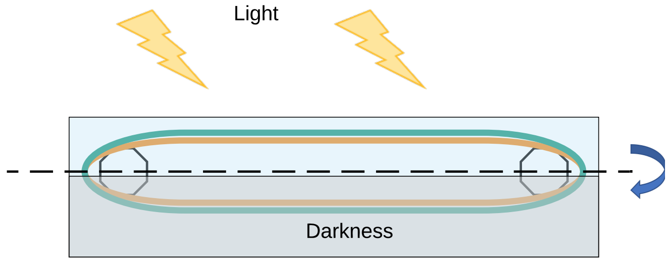

Fig. 1. Schematics of a rotating algal biofilm periodically exposed to dark-light cycles

needs to be achieved for better productivity. We will show through simulation that it is possible to double the productivity. It will also be shown that an optimal value exists, it translates how light needs to be averaged for a given incident light.

The paper is organized as follows:
The different steps in the construction of our model are provided in Section 2. In section 3 the growth rate and productivity are presented to outline light distribution at steady state. Simulation results are found in Section 4. Finally Section 5 summarizes our concluding remarks.

2. MODEL OF THE ROTATING ALGAL BIOFILM

The effect of photo-inhibition is one of the main factors affecting the photosynthetic activity in microalgae, and therefore its growth. The model first introduced in Eilers and Peeters (1988), and then Han (2002) will be used in this paper. They describe the complex process of photosynthesis by two types of photosystems, PSI and PSII. The later will play a prominent role. It is described by three possible states. Open and ready for harvest (A). Closed (activated (B)), and finally the inhibited state C, when many photons are absorbed. The dynamics of the introduced states is given as follows

$$\frac{dA}{dt} = -\sigma I A + \frac{B}{\tau} \tag{1}$$
$$\frac{dB}{dt} = \sigma I A - \frac{B}{\tau} + k_r C - k_d \sigma I B \tag{2}$$
$$\frac{dC}{dt} = -k_r C + k_d \sigma I B, \tag{3}$$

where the A, B and C represent relative frequencies that where A, B, C are the relative frequencies of the three possible states. They satisfy satisfy

$$A + B + C = 1 \tag{4}$$

σ: The effective absorption cross-section unit of photosynthetic units (PSU) $[m^2/\mu E]$
I: represent the light intensity $[\mu mol.m^{-2}.s^{-1}]$
τ: The turnover time of the electron transport chain $[s]$
k_d: The damage rate at which to transit to the inhibition state
k_r: repair rate of damaged photosynthetic units.

Given equation (4) it is possible to simplify the Han model as follows

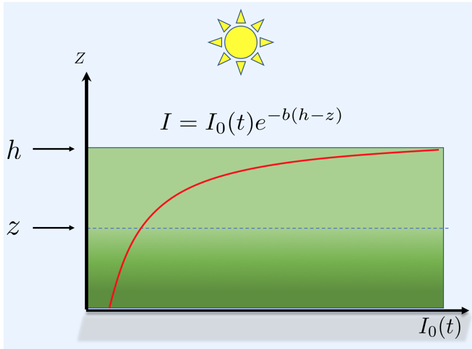

Fig. 2. Light attenuation accross the biofilm layers

$$\frac{dA}{dt} = -\left(\sigma I + \frac{1}{\tau}\right) A + \frac{1-C}{\tau} \tag{5}$$
$$\frac{dC}{dt} = k_d \left(-\left(\frac{k_r}{k_d} + \sigma I\right) C + \sigma I(1-A)\right) \tag{6}$$
$$= -(k_r + k_d \sigma I)C + k_d \sigma I(1-A) \tag{7}$$

The model response for low light frequencies, that is, assuming a slow fast dynamics using singular perturbation theory ($T > 10\tau \approx 1\,min$), A reaches rapidly its pseudo steady state defined by $\dot{A} = 0$ (Hartmann et al. (2014)), therefore the Han model reduces to the following equation

$$\frac{dC}{dt} = -\left(k_d \tau \frac{(\sigma I)^2}{\tau \sigma I + 1} + k_r\right) C + \frac{\tau k_d (\sigma I)^2}{\tau \sigma I + 1} \tag{8}$$

When A reaches steady state we have

$$A = \frac{1-C}{1+\tau\sigma I} \tag{9}$$

2.1 light distribution through the biofilm

Knowing that the biofim's width will increase with time, it is preferable to include the vertical distribution. Here we consider a Beer-Lambert exponential decrease. i.e., whether the RAB is exposed to daylight illumination or to artificial light, it is important to consider the effect of light attenuation through the different layers of the biofilm. Light is distributed through the biofilm according to the following equation

$$I(z) = I_0 e^{-b(h-z)}, \tag{10}$$

where I_0 is the light intensity received on the surface and b is given in (m^{-1}) and b represent the extinction coefficient. From Fig. 2, it is observed that at the bottom $z = 0$, the light is the most attenuated in opposit to the light on the surface when $z = h$. The cycle T correspond to the time the RAB completes one full rotation. T^* is the time the biofilm is exposed to light. The light received by the biofilm is therefore expressed as follows

$$I(t,z) = \begin{cases} I(z) & if \quad 0 \le t \le T^* \\ 0 & if \quad T^* \le t \le T \end{cases} \tag{11}$$

2.2 Mean gowth rate

As reported in Huisman and Weissing (1994), an accurate way of estimating the growth rate in a photobioreactor consists in taking the mean value accross the depth of the photobioreactor. Here the depth average is taking across the width of the biofilm. The specific growth rate is expressed as the balance between photosynthesis and respiration rate, i.e.,

$$\mu = k\sigma I A - R, \tag{12}$$

where k is a constant parameter. Given equation (9), equation (12) becomes

$$\mu(I) = -\frac{k\sigma I}{1+\tau\sigma I}C + \frac{k\sigma I}{1+\tau\sigma I} - R, \tag{13}$$

The mean growth rate across the biofilm width is expressed as follows

$$\bar{\mu}(t) = \frac{1}{h}\int_0^h (1 - C(t,z))\left[\frac{k\sigma I_0 e^{-b(h-z)}}{1+\tau\sigma I_0 e^{-b(h-z)}}\right]\, dz - R \tag{14}$$

2.3 Biofilm width dynamics

Given the surface biomass dynamics $\dot{X} = \bar{\mu}X$ and knowing that $X = \rho h$, where $\rho[g.m^{-3}]$ is the biofilm density, it is possible to express the width dynamics according to

$$\dot{h} = \int_0^h (1 - C)\left[\frac{k\sigma I_0 e^{-b(h-z)}}{1+\tau\sigma I_0 e^{-b(h-z)}}\right] dz - Rh(t), \tag{15}$$

The inhibition dynamics varies according to the following equation

$$\dot{C}(t,z) = -(\beta(t,z) + k_r)C + \beta(t,z) \tag{16}$$

The function $\beta(z)$ is given as

$$\beta(t,z) = \frac{k_d\tau\sigma^2 I(t,z)^2}{\tau\sigma I(t,z) + 1} \tag{17}$$

2.4 Structure of the RAB

In order to improve the productivity of the biofilm It is possible to consider multiple configuration of the RAB's structure. How light is distributed across the biofilm is often measured according to the light dilution factor (LDF), that we also note by $\frac{l_{tot}}{l^*} = N$, where l_{tot} is the total length of the biofilm and l^* is the length of the conveyor belt (biofilm) exposed to light ($N = 2$ in Fig. 1)

2.5 Productivity

In order to assess the efficiency of the process we evaluate the depth and time averaged productivity that is given in terms of $g\, m^{-2}d^{-1}$.

Definition 1. The productivity of the RAB per illuminated area is given by

$$P = \frac{NS}{T_f h}\int_0^{T_f}\int_0^h \left((1 - C(t,z))\phi(h,z) - R\right) X dz dt, \tag{18}$$

It can also be written as follows

$$P = NS\frac{\rho}{T_f}\left(h(T_f) - h(0)\right), \tag{19}$$

where $\rho[g.m^{-3}]$ is the dry biomass density and $T_f = t_h$ is the harvest time. We multiply by NS since we are interested in the productivity per unit of illuminated surface S. Note that $X = \rho h[g.m^2]$ and

$$\dot{h} = \int_0^h \left((1 - C(t,z))\phi(h,z) - R\right) dz dt \tag{20}$$

The function ϕ is given by

$$\phi(h,z) = \frac{k\sigma I_0 e^{-b(h-z)}}{1+\tau\sigma I_0 e^{-b(h-z)}} \tag{21}$$

Note that I_0 can be considered constant or varying depending on the origin of the light source (Day light or constant artificial light).

2.6 Modeling dark-light cycles

Remark 1. Assuming that the length exposed to light l^* is known, that is, the LDF is known, varying the cycle time T would be equivalent to varying the RAB speed.

Note that $T^* = \frac{l^*}{v}$ and $T = \frac{l_{tot}}{v}$, where v is the rotational speed.

The succession of light-dark cycle seen in equation (11) can be modeled according to a periodic square wave function with duty cycle given by the second argument of *sqwave*. The duty cycle is defined as the percent of the signal period in which the square wave is equal to one. It is also equivalent to the exposure time T^*. The light received by the biofilm $I(t)$ can be expressed as follows

$$I(t) = I_0(t) * sqwave\left(\frac{2\pi}{T(t)} * t, \frac{1}{LDF}\right), \tag{22}$$

where I_0 is the light source applied from above to the process in Fig. 1. The presence of time and space variable t and z, respectively, would suggest the use of partial differential equations (Lamare et al. (2017)), however to keep the modeling simple for future control and estimation purposes, we arrived at the following approximation for the dynamics of the RAB.

That the dynamics of the rotating algal biofilm in Fig. 1 can be approximated by the following ode's

$$\dot{h} = \sum_{i=1}^{p}(1 - C_i(t))\left[\frac{k\sigma\bar{I}_i(t)}{1+\tau\sigma\bar{I}_i(t)}\right] - Rh(t) \tag{23}$$

$$\dot{C}_1 = -(\beta_1(\bar{I}_1) + k_r)C_1(t) + \beta_1(\bar{I}_1) \tag{24}$$

$$\vdots \qquad \vdots$$

$$\dot{C}_p = -(\beta_p(\bar{I}_n) + k_r)C_p(t) + \beta_p(\bar{I}_p), \tag{25}$$

where p is the number of discretization layers. It also corresponds to the number of inhibitions ode's.

$$\beta_i(\bar{I}_i) = \frac{k_d\tau\sigma^2\bar{I}_i^2}{\tau\sigma\bar{I}_i + 1} \tag{26}$$

and

$$\bar{I}_i(t) = \frac{4}{h}\int_{z_i}^{z_{i+1}} I(t)e^{-b(h-z)}dz, \tag{27}$$

where $z_i = 0, \frac{h}{p}, \frac{2h}{p}, \ldots, \frac{(p-1)h}{p}$ for $i = 1, 2, 3, \ldots, p-1$ respectively and $z_p = h$.

The previous approximation is justified by the biofilm being divided into p layers. Each layer is associated with its average light. By replacing I in equation (8) by $\bar{I}_1, \bar{I}_2, \ldots, \bar{I}_p$. The inhibition states associated with each layer follows as $C_1, C_2, \ldots, C_p$ and hence equation (24)-(25). Note that the solution of the inhibition states is independent of the variable z. the biofilm's thickness time evolution is given by equation (15). According to the way we divided the different layers, the latter can be written as follows

$$\dot{h} = \sum_{i=1}^{p-1} \int_{z_i}^{z_{i+1}} (1 - C_i(t)) \left[\frac{k\sigma I(t) e^{-b(h-z)}}{1 + \tau\sigma I(t) e^{-b(h-z)}} \right] dz - Rh \tag{28}$$

Using equation (27), the final width dynamics in equation (23) is obtained.
For a number of layer equal to $p = 4$, The biofilm is divided as in Fig. 3

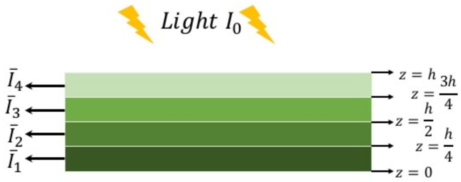

Fig. 3. Biofilm divided into 4 layers. For each separate layer corresponds an average light intensity

3. LIGHT DILUTION AND PRODUCTIVITY

We are interested in this section to see how the distribution of light affects the productivity of the RAB for the case where the conveyor belt of our process is in standstill position, i.e., $N = 1$ or when it is moving very fast. Let us consider eq. (16) and (17). For the scenario where the RAB moves fast, we assume that the variation rate of light is much faster than the dynamics of C (typically minutes) so that averaging can be applied (Verhulst (2006)). Note that we still consider that the dynamics of A (with typical time constants in the range of ms), still more rapidly reaches its steady state.

$$\dot{C} = -(\bar{\beta}(z) + k_r)C + \bar{\beta}(z) \tag{29}$$

$$\bar{\beta}(z) = \frac{1}{T} \int_0^T \beta(t, z) dt \tag{30}$$

Using equation (11) and knowing that $T = NT^*$, we can show

$$\bar{\beta}(z) = \frac{1}{N} \beta(z) \tag{31}$$

3.1 Growth rate

At steady state the growth rate is expressed as follows

$$\mu_{ss}(z) = (1 - C_{ss}(z)) \frac{1}{T} \int_0^T \frac{k\sigma I(t, z)}{1 + \tau\sigma I(t, z)} - R, \tag{32}$$

Similarly to the above discussion, we can write

$$\mu_{ss}(z) = (1 - C_{ss}(z)) \frac{k\sigma I(t, z)}{N(1 + \tau\sigma I(t, z))} - R \tag{33}$$

The photo-inhibition at steady state is given according to

$$C_{ss}(z) = \frac{\beta(z)}{\beta(z) + Nk_r} \tag{34}$$

One can verify that

$$1 - C_{ss}(z) = \frac{Nk_r}{Nk_r + \frac{k_d \tau \sigma^2 I(z)^2}{1 + \tau\sigma I(z)}} \tag{35}$$

Finally after some algebraic simplification the growth rate simplifies to

$$\mu_{ss}(z) = \frac{k k_r \sigma I(z)}{k_d \tau \sigma^2 I(z)^2 + Nk_r(1 + \tau\sigma I(z))} - R \tag{36}$$

Integrate through the width of the biofilm to have the following mean growth rate

$$\bar{\mu}_{ss} = \frac{1}{h} \int_0^h \frac{k k_r \sigma I_0 e^{-b(h-z)} dz}{k_d \tau \sigma^2 I_0^2 e^{-2b(h-z)} + Nk_r \tau\sigma I_0 e^{-b(h-z)} + Nk_r} - R \tag{37}$$

Dividing by $kk_r\sigma$ to have

$$\bar{\mu}_{ss}(h) = \int_0^h \frac{I_0 e^{-b(h-z)}}{\frac{k_d\tau\sigma}{kk_r} I_0^2 e^{-2b(h-z)} + \frac{\tau}{k} I_0 N e^{-b(h-z)} + \frac{N}{k\sigma} dz} - R \tag{38}$$

$$= \frac{1}{b} \int_{J(0)}^{J(h)} \frac{dz}{a_1 J(z)^2 + a_2 J(z) + a_3} - R \tag{39}$$

where we have used the following change of variable

$$\begin{aligned} J(z) &= I_0 e^{-b(h-z)} & a_1 &= \frac{k_d\tau\sigma}{kk_r} \\ dJ(z) &= bJ(z)dz & a_2 &= N\frac{\tau}{k} \\ J(0) &= I_0 e^{-b(h-0)} & a_3 &= \frac{N}{k\sigma} \\ J(h) &= I_0 \end{aligned} \tag{40}$$

$$\begin{aligned} &\int_{J(0)}^{J(h)} \frac{dz}{a_1 J(z)^2 + a_2 J(z) + a_3} \\ &= \frac{2}{\sqrt{4a_1a_3 - a_2^2}} \arctan \frac{2a_1 J + b_2}{\sqrt{4a_1a_3 - a_2^2}} \Bigg|_{J(0)}^{J(h)} \\ &\text{for } 4a_1a_3 - a_2^2 > 0 \\ &= \frac{1}{\sqrt{a_2^2 - 4a_1a_3}} \ln \left| \frac{2a_1 J + a_2 - \sqrt{a_2^2 - 4a_1a_3}}{2a_1 J + a_2 + \sqrt{a_2^2 - 4a_1a_3}} \right| \Bigg|_{J(0)}^{J(h)} \\ &\text{for } 4a_1a_3 - a_2^2 < 0 \\ &= -\frac{2}{2a_1 J + a_2} \Bigg|_{J(0)}^{J(h)} \quad \text{for } 4a_1a_3 - a_2^2 = 0 \end{aligned} \tag{41}$$

so basically the resulting integral is a function of $J(h) = I_0$ and $J(0) = I_0 e^{-bh}$. Let $A_1 = 2a_1 J(0) + a_2$, $A_2 = 2a_1 J(h) + a_2$, and $\Delta = 4a_1a_3 - a_2^2$ then

$$\begin{aligned} \bar{\mu}_{ss}(h) &= \frac{2}{\Delta} \arctan \frac{A_2}{\sqrt{\Delta}} - \frac{2}{\Delta} \arctan \frac{A_1}{\sqrt{\Delta}} \quad if \quad \Delta > 0 \\ &= \frac{2}{A_1} - \frac{2}{A_2} \quad if \quad \Delta = 0 \\ &= \frac{1}{\sqrt{\Delta}} \ln \left| \frac{A_2 - \sqrt{\Delta}}{A_2 + \sqrt{\Delta}} \frac{A_1 + \sqrt{\Delta}}{A_1 - \sqrt{\Delta}} \right| \quad if \quad \Delta < 0 \end{aligned}$$

3.2 Productivity at steady state

Now that we have an expression of the steady state mean growth rate, the productivity per unit of enlightened surface is given as follows

$$P = \bar{\mu}_{ss}(h)XSN, \quad (42)$$

where S is the surface exposed to light. X = ρh, expressed in units of $g/(m^2)$, and $\rho[g.m^{-3}]$ is the dry biomass density. In next section the relationship between productivity and the light dilution factor is going to be outlined.

Table 1. Han's model parameters (Wu and Merchuk (2001))

Parameter	Value	Units
k_r	4.8x10^{-4}	s^{-1}
k_d	2.99x10^{-4}	-
τ	6.849	s
σ	0.0019	$(\mu mol)^{-1}.m^2$
k	3.6467x10^{-4}	

4. SIMULATION RESULTS

The Han parameters introduced in section 2 are provided by Wu and Merchuk (2001), for marine algae Perphyridium sp. The light attenuation factor b is chosen to vary between 500 and 2000. One can observe that $b = 1500$ corresponds to 5% of incident light received at the bottom of the biofilm for a width $h = 2mm$. We choose the respiration $R = 0.12d^{-1}$. The surface exposed to light is assumed to be $S = 1\,m^2$. The dry biomass density $\rho = 140000\,g.m^{-3}$ (Blanken et al. (2014)). The Han parameters used in the simulation are summarized in Table 1. The incident light $I_0 = 2000\,\mu mol.m^{-2}s^{-1}$. Fig.4 shows the evolution of productivity in function of

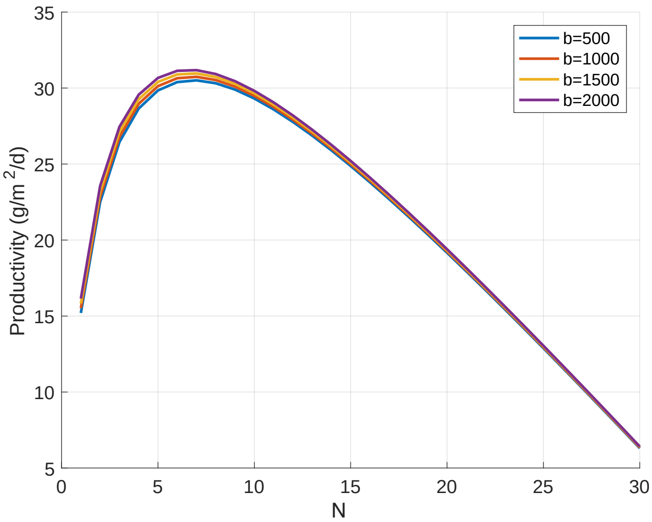

Fig. 4. Productivity variations in function of LDF

the light dilution factor. For $N = 1$, it is shown that the productivity is around $p = 15g.m^{-2}.d^{-1}$. An increase in N is followed by an increase in productivity, to reach a value corresponding to almost the double of the initial one for $N = 7$. Thereafter, the productivity decreases to be negative for very large values of N. This behaviour confirms the hypothesis we have presented in section 3, that is, a better distribution of light would result in productivity increase. The value $N = 7$ represent the good trade-off. It is also the optimal value for maximizing productivity . This is equivalent to say that on average the biofilm is receiving

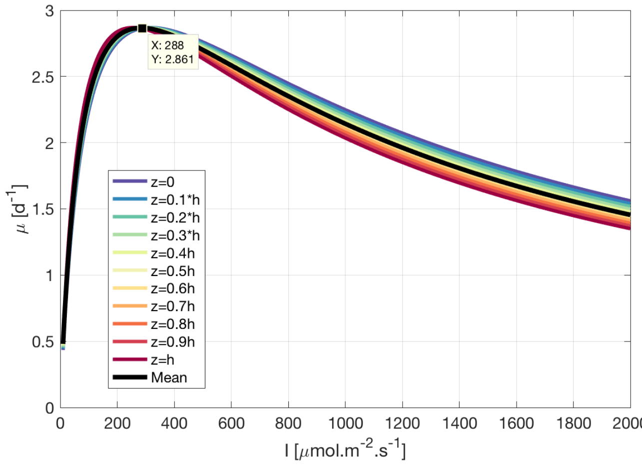

Fig. 5. Growth rate at steady state for various values of incident light. ($h = 110\mu m$, $N = 1$)

$I_0/N \approx 286\mu mol.m^{-1}s^{-1}$. In contrast, the decreasing part of Fig. 4 is due to the fact that an increase in N will correspond to a decrease of the biofilm's part exposed to light. Remember that the total length of the biofilm can be expressed by $l_{tot} = Nl^*$. Therefore, this means that most of the biofilm start spending more time in darkness and the effect of respiration becomes predominant.
We want to evaluate the optimal light intensity when the RAB is permanently exposed to light. That is, the value that will maximize the mean growth rate at steady state. In Fig. 5 growth rate at steady state at different levels of the biofilm are shown. It can be observed that the light intensity that provided the maximum mean growth rate is $I = 288\mu mol.m^{-1}s^{-1}$. Note that this value correspond to approximately the same value as the one for the averaged light. We conclude by saying that in order to increase the productivity the RAB needs to receive in average a light intensity close to $I = 288\mu mol.m^{-1}s^{-1}$.

5. CONCLUSION

Processes based on biofilms solve the drawbacks that suspended microalgae culture experience in terms of energy and time. A mathematical model for a rotating algal biofilm process is presented. It outlines the positive effect that light can have on maintaining a positive biofilm growth thanks to photosynthesis. In contrast, photoinhibition acts in the opposite direction. Light attenuation through the biofilm layers is modeled by the Beer-Lambert effect. Dividing the biofilm in multiple layers and taking the average light intensity for each part, allows to have a relatively good approximation. We have shown that for high light intensity, it is possible to counteract the effect of inhibition by averaging light. In fact, it has been possible to double the productivity by choosing the optimal LDF. It turns out that maximum productivity occurs when the averaged incident light is close to the optimal value that maximizes the mean growth rate when the RAB is fully exposed to light.

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